EXERCISES:

31. A 3.50 mL chunk of boron has a mass of 8.19 g. What is the density of the boron?
32. An iron bar has a mass of 125 g. If iron's density is 7.86×10^3 g/L, what volume does the bar occupy ?
33. A block of beeswax has a volume of 200.0 mL and a density of 961 g/L. What is the mass of the block?
34. Alcohol has a density of 789 g/L. What volume of alcohol is required in order to have 46 g of alcohol?
35. A gas called neon is contained in a glass bulb having a volume of 22.4 L. If the density of the neon is 0.900 g/L, what is the mass of the neon in the bulb?
36. A 70.0 g sphere of manganese (density = 7.20×10^3 g/L) is dropped into a graduated cylinder containing 54.0 mL of water. What will be the water level indicated after the sphere is inserted?
37. A 25.0 mL portion of each of W, X, Y and Z is poured into a 100 mL graduated cylinder. Each of the 4 compounds is a liquid and will not dissolve in the others. If 55.0 mL of W have a mass of 107.3 g, 12.0 mL of X have a mass of 51.8 g, 42.5 mL of Y have a mass of 46.8 g and 115.0 mL of Z have a mass of 74.8 g, list the layers in the cylinder from top to bottom.
38. Explain why boats made of iron are able to float. The density of iron is 7.86×10^3 g/L.
39. If the density of copper is 8.92×10^3 g/L and the density of magnesium is 1.74×10^3 g/L, what mass of magnesium occupies the same volume as 100.0 g of copper?
40. The sun has a volume of 1.41×10^{30} L, an average density of 1.407 g/mL, and can be thought of as more or less pure hydrogen. If the sun consumes 4.0×10^6 t of hydrogen per second, how many years will it take at this rate to burn all of the hydrogen? Hint: use the results of exercise 17(k). The sun will actually cease burning its hydrogen in far less time than indicated by this simple calculation.
41. (OPTIONAL: A Stinker!) A hollow cylinder, closed at both ends, has a volume of 250.0 mL and contains 4.60 g of argon gas. A 90.0 g cube of sodium (density = 970.0 g/L) is inserted into the tube in such a way that no gas escapes. What is the density of the gas afterwards?

II.5. SIGNIFICANT FIGURES AND EXPERIMENTAL UNCERTAINTY

When **COUNTING** a small number of objects it is not difficult to find the **EXACT** number of objects. On the other hand, when a property such as mass, time, volume or length is **MEASURED** you can ***never*** find the exact value. It is possible to find a mass, say, very precisely but it is ***impossible*** to find an object's exact mass.

All measurements have a certain amount of "uncertainty" associated with them. The purpose of this section is to show you how to correctly report and use the results of the experimental measurements you will be making in Chemistry 11.

You will need to learn
(a) how to find and report the uncertainty associated with each measurement, and
(b) the number of digits which can be claimed when reporting results and carrying out calculations with the results.

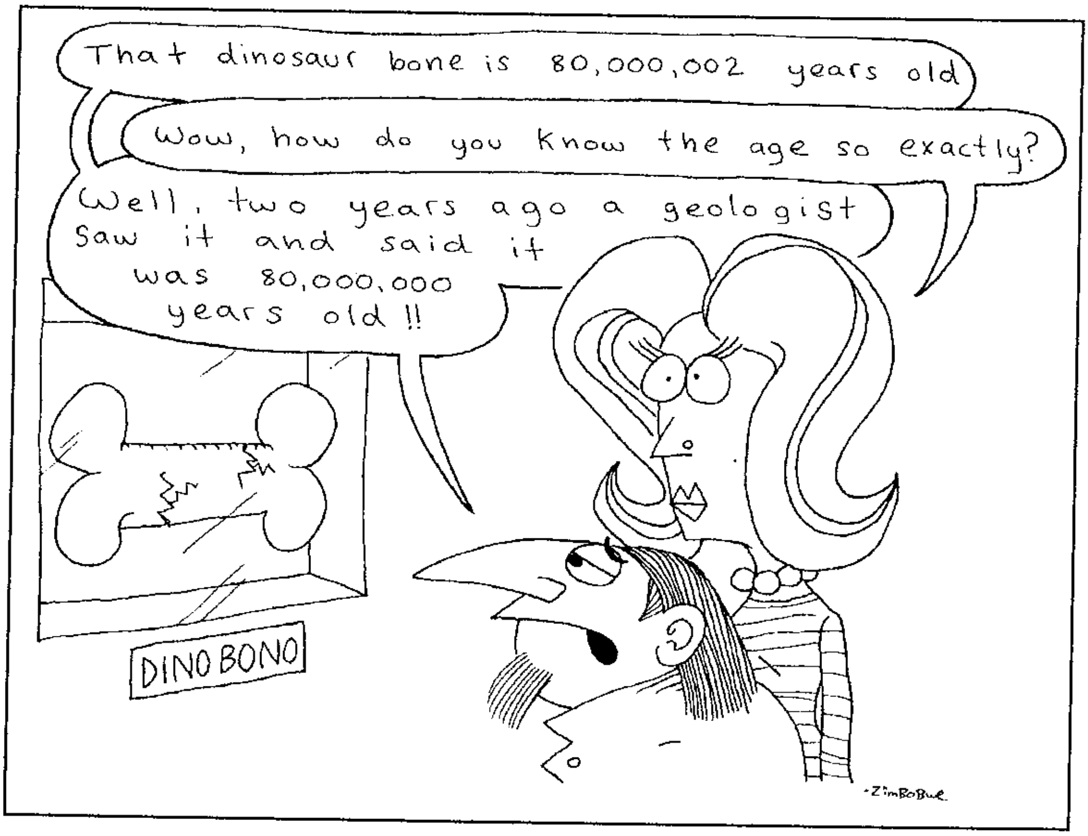
Let's look at the rules of the game.

SIGNIFICANT FIGURES

A. **A significant figure** is a **measured** or **meaningful digit.**

EXAMPLE: If a stopwatch is used to time an event and the elapsed time is 35.2 s, then the measurement has 3 significant figures (3, 5 and 2). If the stopwatch can only be read to 0.1 s then it is silly to claim that the time according to the stopwatch is 35.**2168497** s. Since the stopwatch cannot measure the time to 7 decimal places, the last digits (**168497**) have **no significance** – in other words these last digits are "imagined" or a joke.

EXAMPLE: A balance gives a reading of 97.53 g when a beaker is placed on it. This first reading has 4 significant figures since the measurement contains 4 digits. The beaker is then put on a different balance, giving a reading of 97.5295 g. In this second case there are more significant figures to the measurement (6 significant figures).



SPECIAL NOTE: When a ***measurement*** is reported it is usual to assume that in numbers such as

10, 1100, 120, 1000, 12 500

any zeroes at the end are NOT SIGNIFICANT WHEN NO DECIMAL POINT IS SHOWN. That is, we assume the last digits are zeroes because they are rounded off to the nearest 10, 100, 1000, etc. The number of significant digits in the above examples are shown in parentheses, below.

10 (1), 1100 (2), 120 (2), 1000 (1), 12 500 (3)

To complicate matters, SI usage dictates that a decimal point cannot be used without a following digit. For example, 10.0 and 100.0 are legal examples of SI usage with 3 and 4 significant digits respectively, but 10. and 100. are "illegal" ways of showing numbers. If you need to show that a number has been measured to 3 significant figures and has a value of 100 or that the number 1000 has actually been measured to 4 significant figures, the solution is to use exponential notation.

$1.00 \times 10^2 = 100$ (to 3 significant figures)
$1.000 \times 10^3 = 1000$ (to 4 significant figures)

EXERCISE:

42. How many significant figures do each of the following measurements have?

(a) 1.25 kg	(c) 11 s	(e) 1.283 cm	(g) 2 000 000 years
(b) 1255 kg	(d) 150 m	(f) 365.249 days	(h) 17.25 L

B. An **ACCURATE** measurement is a measurement that is close to the CORRECT or ACCEPTED value. (The closer to the correct/accepted value, the more accurate the measurement.)

A **PRECISE** measurement is a reproducible measurement. In general, the more precise a measurement, the more SIGNIFICANT DIGITS it has.

Notes: 1. The accuracy of a measuring instrument depends on whether the instrument is properly "calibrated". For example, if a reference mass of 50.000 g is put on an electronic balance and the balance gives a reading of 48.134 g, the balance is not accurate. A special adjustment on the electronic balance is then used to make the balance give a reading of 50.000 g, in agreement with the reference mass. This adjustment process is called a "calibration".

2. For the purposes of Chemistry 11, "high precision" shall be used to mean a "high number of significant figures". In general, it is reasonable to assume that if an instrument in good operating condition can give a reading to 8 significant figures a first time, it will give the same (and therefore reproducible) measurement a second time – provided what is being measured does not change. (There are cases where an instrument can record many digits but not give reproducible results as the result of "machine malfunction" or random errors, but these are ignored for our purposes.)

EXAMPLES: Assume the **CORRECT** width of a room is 5.32000 m.

- A measurement of 5.3 m is **ACCURATE but not VERY PRECISE.** (The value "5.3" is very close to correct as far as its significant figures go, but there are not many significant figures so the value is not very precise)
- If several measurements with some device consistently give the width as 5.45217 m, the measurements are **PRECISE but not ACCURATE.** (Apart from the initial "5" none of the significant digits are correct, so the measurements lack accuracy. The measurements DO have several significant figures, however, so they are precise.)
- If a measurement is consistently given as 5.32001 m, it is **ACCURATE and PRECISE.** (The measured value has many significant figures, so it is precise, and the measured digits agree very well with the correct value, so the measurement is accurate.)
- If a measurement is 7.1 m, it is **not ACCURATE and not PRECISE.** (There are very few significant figures, so the measurement is not very precise, and all the digits are in error so the measurement is not accurate.)

EXERCISES:

43. Assume you have a balance which gives very precise measurements. What might be true about the balance in order that its readings would be precise but not accurate?

44. A "calibration weight" has a mass of exactly 1.000 000 g. A student uses 4 different balances to check the mass of the weight. The results of the weighings are shown below.

    mass using balance A = 0.999 999 g   mass using balance C = 3.0 g
    mass using balance B = 1.00 g   mass using balance D = 0.811 592 g

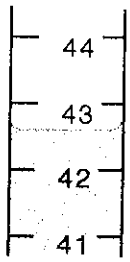
    (a) Which of the balances give accurate weighings?
    (b) Which of the balances give precise weighings?
    (c) Which balance is both accurate and precise?

45. An atomic clock is used to measure a time interval of 121.315 591 s. Assume you have to measure the same time interval. Give an example of a time interval you might actually measure if your measurement is:

    (a) not accurate, but is precise.
    (b) not precise, but is accurate.
    (c) both inaccurate and imprecise.
    (d) both accurate and precise.

C. The **number** of significant figures is equal to all the **certain** digits PLUS the **first uncertain** digit.

EXAMPLE: In the figure at the right, the liquid level is somewhere between 42 and 43 mL. You know that it is at least 42 mL, so you are "certain" about the first two digits. As a guess, the volume is about 42.6 mL; it could be 42.5 or 42.7 but 42.6 seems reasonable. There is some "significance" to this last, guessed digit. It is somewhat uncertain, ***but*** not completely so. For example, the reading is NOT 42.1 or 42.9. As a result, there are two CERTAIN digits (4 and 2) and one uncertain–but–still–significant digit (6) for a total of THREE significant figures.



NOTE: If you are given a measurement without being told something about the device used to obtain the measurement, assume that the LAST DIGIT GIVEN IS SOMEWHAT UNCERTAIN.

EXERCISE:

46. How many "certain" digits are contained in each of the following measurements?
    (a) 45.3 s   (b) 125.70 g   (c) 1.85 L   (d) 2.121 38 g

D. "**Defined**" numbers and "**counting**" numbers are assumed to be **PERFECT** so that they are "exempt" from the rules applying to significant figures.

Defined or counted values involve things which cannot realistically be subdivided and must be taken on an "all–or–nothing" basis.

EXAMPLES: When "1 book" or "4 students" is written, it means exactly "1 book" and "4 students", not 1.06 books and 4.22 students.

The conversion factor 1 kg = 1000 g is used to define an exact relationship between grams and kilograms, so that the numbers involved are assumed to be perfect.

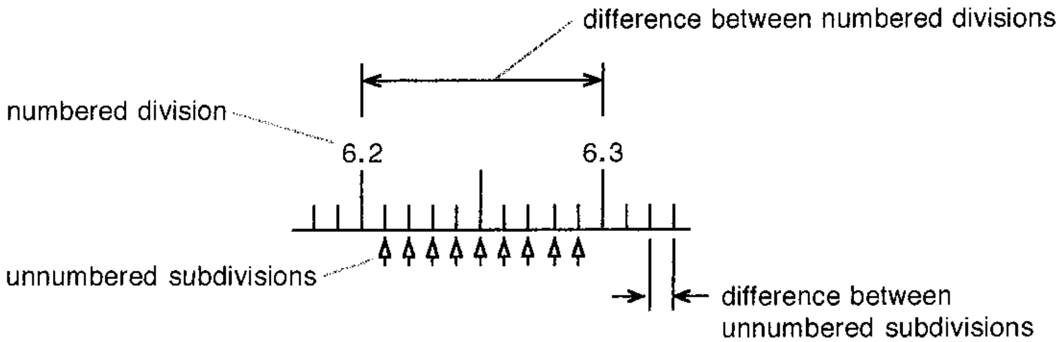
EXERCISE:

47. In the space following each value below put "M" if the value was likely obtained by a Measurement, or "C" if the value was probably determined by Counting.
    (a) 4 comets   (b) 45 seconds   (c) 6.5 litres   (d) 12 TV sets   (e) 12 grams

HOW TO READ A SCALE

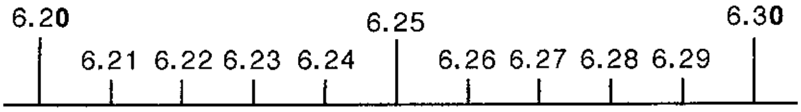
Before learning more about uncertainty, you must first be able to read a scale properly.

IMPORTANT: The following terms are used –



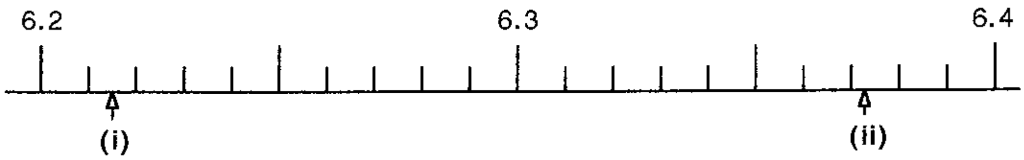
Both the numbered divisions and unnumbered subdivisions are **CALIBRATED** DIVISIONS because the overall scale has been "marked off" or "calibrated" at regular intervals.

If the unnumbered subdivisions ***were numbered***, they would be labelled as shown below.



The "numbered divisions" would then read "6.20" and "6.30", rather than "6.2" and "6.3". The unnumbered subdivisions allow two more decimal places to be read. For example, the numbered divisions above differ in the first decimal place and the unnumbered subdivisions allow a reading to the second decimal place. The estimated distance between unnumbered subdivisions allows a reading to the third decimal place.

EXAMPLES: **(a) What is the value of (i) and (ii) on the following centimetre scale?**



The first two digits of (i) are 6.2 and the first two digits of (ii) are 6.3. The problem is to read the next two digits for each point.

FIRST: Find the difference between each NUMBERED DIVISION.
In the above example: 6.3 – 6.2 = 0.1 cm.

SECOND: Find the number of unnumbered subdivisions between numbered divisions and calculate the value of each unnumbered subdivision. Each numbered division above has 10 subdivisions and each unnumbered sub–division is

$$\frac{0.1\text{ cm}}{10} = 0.01\text{ cm}.$$

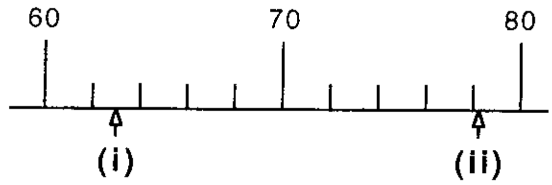
Since the unnumbered subdivisions have a value of 0.01 cm, the value at (i) is a little more than 6.21 cm and the value at (ii) is a little more than 6.37 cm.

THIRD: Estimate how far along their respective unnumbered subdivisions (i) and (ii) are; this gives a reading to the next decimal place, which is the uncertain digit.

Reading at (i): The 3 certain digits are "6.21". The pointer is half–way from 6.21 to 6.22, so the uncertain 4th digit is probably a "5". Therefore, the reading is **6.215 cm.**

Reading at (ii): The 3 certain digits are "6.37". The pointer is $^3/_{10}$ of the way from 6.37 to 6.38, so the uncertain 4th digit is probably a "3". Therefore, the reading is **6.373 cm.**

(b) What is the value of (i) and (ii) on the following centimetre scale?



The value of (i) lies between 60 and 70 cm; the value of (ii) lies between 70 and 80 cm.

FIRST: The difference between numbered divisions is 10 cm.

SECOND: There are 5 subdivisions between each numbered division, so each unnumbered subdivision is equal to

$$\frac{10\,\text{cm}}{5} = 2\,\text{cm}\ .$$

THIRD: Pointer (i) lies between 62 and 64, and pointer (ii) is between 78 and 80.

Reading at (i): The pointer is about half–way ($^5/_{10}$) between 62 and 64. Therefore the reading is more than 62 cm by $^5/_{10}$ of 2 cm (the subdivision value).

reading = 62 cm + 0.5 x 2 cm = **63.0 cm**

(The numbered divisions differ by "tens", the unnumbered subdivisions are read to the "ones" and an estimate between unnumbered subdivisions are read to "tenths".)

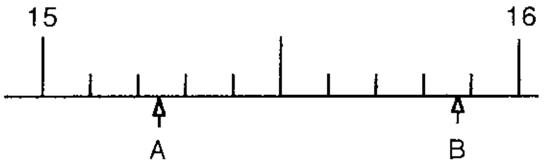
Reading at (ii): The pointer is about $^1/_{10}$ of the way from 78 to 80. Therefore the reading is more than 78 by $^1/_{10}$ of 2 cm.

reading = 78 cm + 0.1 x 2 cm = **78.2 cm**

EXERCISE:

48. In each of the following, determine the reading as follows. ***Note***: all measurements are in "cm".
    i) Find the difference between each numbered division.
    ii) Find how many unnumbered subdivisions lie between each numbered division and calculate the value of the intervals between each unnumbered subdivision.
    iii) Estimate the value at the pointer (you will have to estimate how far the pointer is from one unnumbered subdivision to the next).

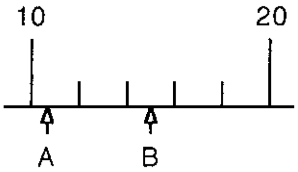
(a)



A = ____________

B = ____________

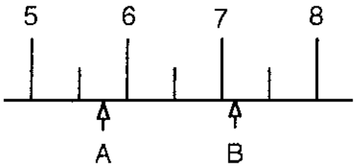
(b)



A = ____________

B = ____________

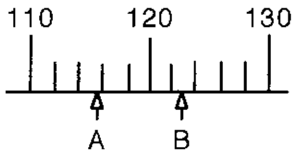
(c)



A = ____________

B = ____________

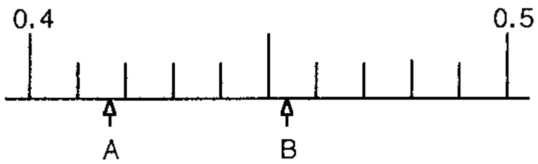
(d)



A = ____________

B = ____________

(e)

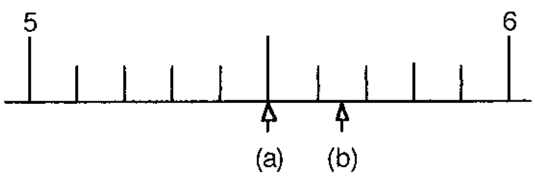


A = ____________

B = ____________

There is one last problem associated with reading scales that must be examined: **what to do when the "pointer" is exactly on one of the markings.**

EXAMPLE: Look at the centimetre ruler and indicated values below.



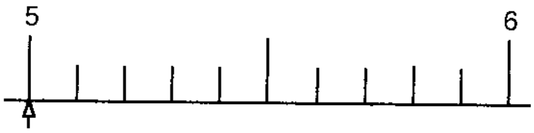
The pointer at (a) seems to indicate a value of 5.5 cm but that is ***not*** the correct value. Look at the pointer at (b). Since the value at (b) is about 5.65, both the value at (a) and (b) can be guessed to the nearest **0.01 cm.** The value for (a) must be given as **5.50 cm.**

BE VERY CAREFUL WHEN A VALUE APPEARS TO COINCIDE EXACTLY WITH A MARKING ON A MEASURING DEVICE. The following procedure should help when such a situation occurs.

THE PROCEDURE FOR CORRECTLY READING MEASURING SCALES WHEN A POINTER IS EXACTLY ON A NUMBERED DIVISION

- Determine the value that the measurement seems to have.
- Pretend you have a value in between two of the unnumbered subdivisions on your measuring device.
- Determine how many decimal places you could read off the measuring device at the "in–between value".
- Add a sufficient number of zeroes to the actual reading to give you the correct number of decimal places for your reading.

In the example above, the intervals between unnumbered subdivisions can be read to 0.01 cm; that is, to 2 decimal places. The reading appears to be 5.5, which is only 1 decimal place, so an extra zero is added to get the value: **5.50 cm.** Similarly, consider the value of the measurement below.

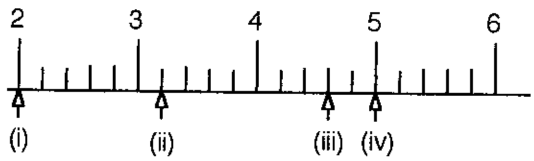


The value seems to be 5 cm, but the previous example shows that an "in–between value" can be read to 0.01 cm (2 decimal places) and so 2 extra zeroes are added to arrive at the final reading: **5.00 cm.**

EXERCISES:

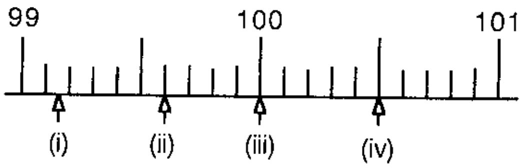
49. Determine the readings on the following centimetre rulers.

(a)



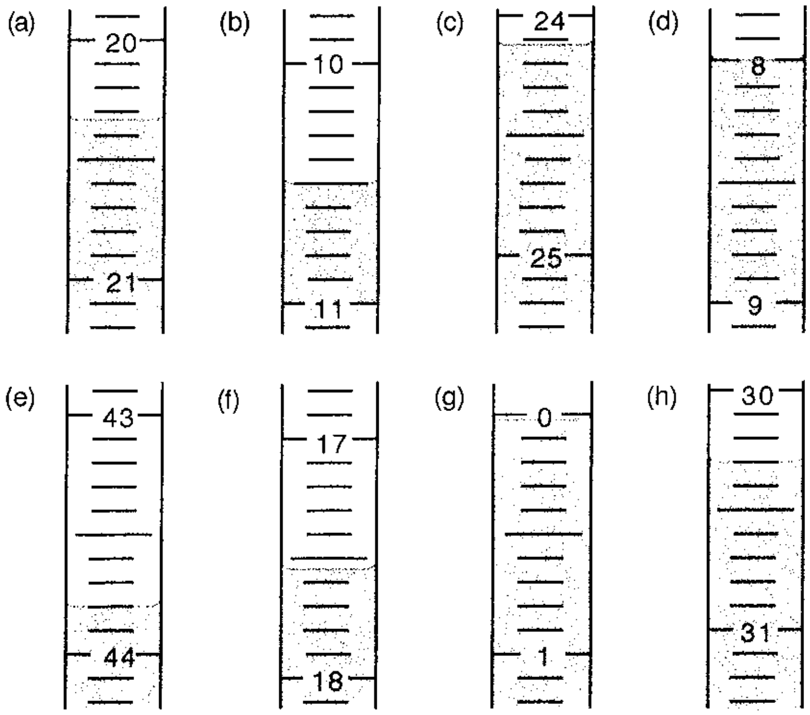
Reading at (i) = ____________
Reading at (ii) = ____________
Reading at (iii) = ____________
Reading at (iv) = ____________

(b)



Reading at (i) = ____________
Reading at (ii) = ____________
Reading at (iii) = ____________
Reading at (iv) = ____________

50. Determine the volume readings of the following burettes. **Care!** The numbers ***increase*** going ***down*** the scale.



EXPERIMENTAL UNCERTAINTY

Having seen how to deal with significant figures and make proper readings, the next step is to learn about experimental uncertainty.

Definition: The experimental uncertainty is the estimated amount by which a measurement might be in error.

E. | When adding an uncertainty to a measurement, the uncertainty goes after the measured value but before the unit. |
|---|

EXAMPLE: Assume that a measured temperature is 39.6°C and the uncertainty in the measurement is ± 0.1°C (Part F shows how to estimate the uncertainty). The measurement and uncertainty are shown below.

$$39.6 \pm 0.1^{o}C$$

certain digits — 39
uncertain digit — 6
uncertainty which is possessed by the uncertain digit — ± 0.1
put the unit at the end — °C

NOTE: If the uncertain digit is in the first decimal place, the uncertainty will be in the first decimal place also.

INTERPRETATION OF UNCERTAINTIES

When a measurement is said to be 39.6 ± 0.1°C, this implies that the actual value most likely lies in the range from (39.6 – 0.1)°C to (39.6 + 0.1)°C; that is, from 39.5°C to 39.7°C.

Similarly the measurement 15.55 ± 0.02 mL implies a volume in the range from 15.55 – 0.02 = 15.53 mL to 15.55 + 0.02 = 15.57 mL.

If only the range of probable values is known, for example 88.0 g to 89.0 g, the uncertainty is simply one–half of the stated range.

$$\text{range} = 89.0 - 88.0 = 1.0\ \text{g}$$
$$\text{uncertainty} = \tfrac{1}{2}(1.0) = 0.5\ \text{g}$$

The measurement reported is the MIDPOINT of the range, plus/minus the uncertainty. The midpoint of the range is simply the AVERAGE of the endpoints of the range.

$$\text{midpoint} = \tfrac{1}{2}(88.0 + 89.0) = 88.5\ \text{g}$$

Therefore, the reported measurement is **88.5 ± 0.5 g.**

Similarly, if the range is 15.0 g to 15.5 g then the midpoint of the range is 15.3 g (to one decimal point) and the uncertainty is ½ (0.5) = 0.3 g (to one decimal point). [Yes, 15.3 ± 0.3 g predicts the range as 15.0 – 15.6 g, but the range, midpoint and uncertainty are all recognized as simply "good guesses".]

IMPORTANT: The place values (tens, units, first decimal, etc.) of the experimental uncertainty and the first uncertain digit of a measurement must agree with each other.

EXAMPLES: 15.5°C ± 0.01°C is ***wrong*** because the measurement is only read to the nearest 0.1°C, which means the first decimal place is uncertain. An uncertainty of 0.01°C implies the measurement can be read with at least partial certainty to the second decimal place.

5.52 ± 0.01 mL is ***correct*** because the last (uncertain) digit in the measurement and the uncertainty quoted are both to the second decimal place.

EXERCISES:

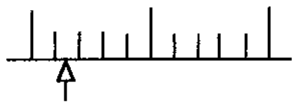
51. Write the following measurements and uncertainties in the correct form.
    (a) A balance gives a mass reading of 51.32 g. The balance has an uncertainty of 0.01 g.
    (b) A student records a volume of 55 mL with an uncertainty of 1 mL.
    (c) A measurement is in the range from 452 g to 458 g.
    (d) A measurement is in the range from 0.5128 g to 0.5132 g.
    (e) Several people time the same event. Their times range from 98.2 s to 99.5 s.
    (f) A series of white mice have masses ranging from 48.9 g to 50.6 g.

52. What is the range of values possible for the following?
    (a) 15.25 ± 0.01 mL (b) 110.0 ± 0.2 mL (c) $1.528 \times 10^{-6} \pm 0.005 \times 10^{-6}$ s

F. **NORMALLY USE UNCERTAINTIES TO THE NEAREST 0.1 OF THE SMALLEST UNNUMBERED SUBDIVISION.** ***If*** you can only estimate a value to the nearest ±0.2 or even ±0.5 of the smallest unnumbered subdivision, feel free to do so, but be prepared to justify your decision. (Sometimes values are hard to read.)

Now that you know HOW to read a scale, estimating the uncertainty is relatively easy.

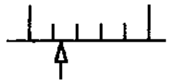
EXAMPLE: Look at the centimetre scale below.



The pointer indicates a value of 6.214, and the last digit ("4") is somewhat uncertain. **The place value (third decimal place) of the experimental uncertainty and the first uncertain digit of a measurement must agree with each other.** Therefore the value and uncertainty are

6.214±0.001 cm .

EXAMPLE: The value on the scale below is 2.26 cm and $^1/_{10}$ of an unnumbered subdivision is 0.02 cm.



Therefore, the value and uncertainty are **2.26 ± 0.02 cm .**

EXERCISES:

53. Record values for the experimental uncertainty encountered in using the following apparatus.

Apparatus	Difference between numbered divisions	# of unnumbered subdivisions between numbered divisions	Smallest unnumbered subdivision	Uncertainty of measurement
thermometer				
10 mL graduated cylinder				
25 mL graduated cylinder				
100 mL graduated cylinder				
250 mL graduated cylinder				
50 mL burette				
clock				
balance				
Other:				
Other:				
Other:				

54. Determine the uncertainty for each of the measurements in exercises 48–50, and record the measurement, uncertainty and units in the correct fashion.

G. **Leading** zeroes are **not** significant.

EXAMPLE: The mass "25 g" has 2 significant figures. Using a unit conversion to express 25 g in kilograms gives

$$\# \text{ of kg} = 25 \text{ g} \times \frac{1 \text{ kg}}{10^3 \text{ g}} = 0.025 \text{ kg} .$$

A more precise measurement was not performed so the measurement must still have two significant figures, the 2 and 5. The leading zeroes (in bold) in **0.0**25 kg are NOT SIGNIFICANT. Notice that re–expressing 25 g in megagrams increases the number of leading zeroes — 0.000 025 Mg — but the leading zeroes are not significant.

The number of leading zeroes depends on the size of the unit used to express the measured value, and is ***not related*** to the precision, accuracy or number of significant figures.

H. **Trailing** zeroes are **all** assumed to be significant and must be justified by the precision of the measuring equipment.

EXAMPLE: The zeroes at the end of the following 2 numbers are called "trailing zeroes".

25.00 g	represents the precision of a common lab balance (4 significant figures in this case)
25.000 000 g	represents a highly precise microbalance (8 significant figures)

A balance precise to at least 0.000 001 g is required in order to ensure that the trailing zeroes in 25.000 000 g are zeroes and not some other digits.

(If a balance capable of making a measurement to ± 0.01 g is used and the result is written as "35.6 g", it is not correct to say "*Oh, I forgot to record the second decimal place, it must have been a zero.*" In fact, any reading from "35.60 g" to "35.69 g" was equally possible. Write "35.60 g" only when the balance actually shows "0" in the second decimal place.)

EXCEPTION: Recall that a number which is written without a decimal and has been rounded off to the nearest 100 (say) does not claim that the last zeroes are significant.

Example: In 38 500 g the trailing zeroes are NOT SIGNIFICANT.

To summarize all the comments on leading and trailing zeroes —

There are two ways to count the number of significant figures.

EXPRESS THE NUMBER IN SCIENTIFIC NOTATION AND THEN COUNT ALL THE DIGITS.

Or even simpler:

starting from the left side of the number, ignore all "leading zeroes" and only start counting at the first NON–ZERO digit. Once you start counting, continue until you run out of digits.

EXAMPLE: 0.000 035 000 = 3.5000×10^{-5}, and therefore the number has 5 significant figures.

EXERCISE:

55. State the number of significant figures in each of the following.

(a) 3570	(c) 41.400	(e) 0.000 572	(g) 41.50×10^{-4}	(i) $1.234\ 00 \times 10^{8}$
(b) 17.505	(d) 0.51	(f) 0.009 00	(h) $0.007\ 160 \times 10^{5}$	(j) $0.000\ 410\ 0 \times 10^{7}$

I. **After MULTIPLYING or DIVIDING numbers, round off the answer to the LEAST NUMBER OF SIGNIFICANT FIGURES contained in the calculation.**

EXAMPLE: 2.00 x 3.000 00 = 6.00

2.00	3.000 00	6.00
↑	↑	↑
3 significant figures	6 significant figures	3 significant figures

Since the calculation involves a lower precision number (3 significant figures) and a higher precision number (6 significant figures), the precision of the result is limited by the LEAST precise number involved. The answer has only 3 significant figures.

When multiplying 2 numbers like 5.0 x 20.0 you must be careful how you write the answer.

5.0 x 20.0 = 100 **IS WRONG!**

In this case a 2 significant figure number (5.0) is being multiplied by a 3 significant figure number (20.0), so the answer is only allowed to have 2 significant figures. Since "100" implies 1 significant figure, you MUST change to EXPONENTIAL FORM to properly show that the answer has 2 significant figures.

$$5.0 \times 20.0 = 1.0 \times 10^2$$

EXAMPLE: $\frac{15.55 \times 0.012}{24.6} = 0.0076$

This example involves numbers with 4, 3 and 2 significant figures. Since the least precise number, 0.012, has only 2 significant figures, the answer is rounded off to 2 significant figures.

EXAMPLE: $\frac{2.4000}{8.000} = 0.3000$

If you perform this calculation on your calculator, the result shown will be "0.3". ***BUT,*** the answer must have 4 significant figures, so three ZEROES are added to indicate that the answer is "0.3000" to 4 significant figures.

EXAMPLE: $\frac{2.56 \times 10^5}{8.1 \times 10^8} = 3.2 \times 10^{-4}$

The exponential parts of the numbers do not contribute to the number of significant figures. This calculation has a 3 significant figure number, 2.56×10^5, divided by a 2 significant figure number, 8.1×10^8. Putting these numbers into a calculator, and rounding the final answer to 2 significant figures, gives 3.2×10^{-4}.

IMPORTANT: YOU MUST **ALWAYS** PERFORM CALCULATIONS TO THE MAXIMUM NUMBER OF SIGNIFICANT FIGURES ALLOWED BY YOUR CALCULATOR AND **ONLY** YOUR FINAL ANSWER SHOULD BE ROUNDED OFF TO THE CORRECT NUMBER OF SIGNIFICANT FIGURES. ROUNDING OFF **INTERMEDIATE** ANSWERS OFTEN PRODUCES INCORRECT RESULTS.

If you cannot keep all your calculated values in your calculator (or its memory), then always round off intermediate results so as to keep at least ONE "SIGNIFICANT FIGURE" more than you will eventually use in your final result.

EXERCISE:

56. Perform the indicated operations and give the answer to the correct number of significant figures.

(a) 12.5 x 0.50
(b) 0.15 x 0.0016
(c) 40.0 / 30.0000
(d) 2.5 x 7.500 / 0.150
(e) $(6.40 \times 10^{8}) \times (5 \times 10^{5})$
(f) $4.37 \times 10^{3} / 0.008\ 560\ 0$
(g) 51.3 x 3.940
(h) $0.51 \times 10^{-4} / 6 \times 10^{-7}$
(i) 4.75 x 5
(j) 0.000 01 / 0.1000
(k) 7.4 / 3
(l) 0.000 43 x 0.005 001

J. After **ADDING** or **SUBTRACTING** numbers, round off the answer to the **LEAST NUMBER OF DECIMAL PLACES** contained in the calculation.

The idea behind this rule is simple. The number with the least number of decimal places is least precise and limits the precision of the final result.

EXAMPLE:

    12.56 cm
  + 125.8 cm
    138.36 cm

The second value, 125.8 cm, is only precise to the1st decimal place so the final answer is rounded off to one decimal place: **138.4 cm.**

EXAMPLE:

    41.037 6 g
  − 41.037 584 g
    0.000 016 g

Since the least precise number has 4 decimal places, the answer must be rounded to 4 decimal places: **0.0000 g.** This answer is interpreted to mean there is no significant difference between the numbers being subtracted.

EXAMPLE: $1.234 \times 10^{6} + 4.568 \times 10^{7} = ?$

Since the exponents are different, one of the exponents must be changed to the size of the other. Arbitrarily, change the smaller exponent so that it equals the larger one.

$\mathbf{1.234 \times 10^{6}}$ becomes $\mathbf{0.1234 \times 10^{7}}$

(Since the exponent becomes one power of 10 larger (10^{6} becomes 10^{7}), the number in front is made one power of 10 smaller to compensate.)

Now the numbers are added.

0.1234×10^{7}
$+ 4.568 \times 10^{7}$
4.6914×10^{7}

Since the second number is only known to the third decimal place (in its present exponential form), the answer is rounded to the third decimal place.

Answer = $\mathbf{4.691 \times 10^{7}}$

Note: When an uncertain number is multiplied or divided by an exact (counting) number, the result obeys the rules for adding or subtracting the uncertain number. In other words, the answer is rounded to the same number of decimal places as the uncertain number.

EXAMPLES: The weights of 3 boys are: 51.0 kg, 52.4 kg and 49.8 kg. The average of their weights is

$$\frac{(51.0 + 52.4 + 49.8)}{3} = \mathbf{51.1\ kg.}$$

DO NOT try to restrict the answer to 1 significant figure, because "3 boys" is an exact (counting) number rather than a "1 significant figure" number.

A Canadian nickel has a mass of 4.53 g. The mass of three such nickels is:
3 x 4.53 = 13.59 g (the "3" is exact; both "4.53" and "13.59" have 2 decimal places)

IN SUMMARY

When ***multiplying*** or ***dividing*** two numbers, the result is rounded to the least number of significant figures used in the calculation.

When ***adding*** or ***subtracting*** two numbers, the result is rounded to the least number of decimal places used in the calculation.

EXERCISES:

57. Perform the indicated operations and give the answer to the correct number of significant figures.
    - (a) 15.1 + 75.32
    - (b) 178.904 56 – 125.8055
    - (c) $4.55 \times 10^{-5} + 3.1 \times 10^{-5}$
    - (d) 0.000 159 + 4.0074
    - (e) $1.805 \times 10^{4} + 5.89 \times 10^{2}$
    - (f) 0.000 048 1 – 0.000 817
    - (g) $7.819 \times 10^{5} - 8.166 \times 10^{4}$
    - (h) 45.128 + 8.501 87 – 89.18
    - (i) $0.0589 \times 10^{-6} + 7.785 \times 10^{-8}$
    - (j) $89.75 \times 10^{-12} + 6.1157 \times 10^{-9}$
58. Perform the indicated operations and give the answer to the correct number of significant figures.
    - (a) 7.95 + 0.583
    - (b) 1.99 / 3.1
    - (c) 4.15 + 1.582 + 0.0588 – 35.5
    - (d) 1200.0 / 3.0
    - (e) $5.31 \times 10^{-4} / 3.187 \times 10^{-8}$
    - (f) 45.9 – 15.0025
    - (g) 375.59 x 1.5
    - (h) $5.1076 \times 10^{-3} - 1.584 \times 10^{-2} + 2.008 \times 10^{-3}$
    - (i) $1252.7 - 9.4 \times 10^{2}$
    - (j) 0.024 00 / 6.000

Mixed calculations involving the addition, subtraction, multiplication and/or division of uncertain values are treated in a step–by–step manner, as shown in the next example.

EXAMPLE: Perform the indicated operations and give the answer to the correct number of significant figures.

50.35 x 0.106 – 25.37 x 0.176 = ?

First: Evaluate the multiplications (and divisions, if present). All digits given by the calculation are shown, with significant figures shown in bold.

50.35 x 0.106 = **5.337**1
25.37 x 0.176 = **4.465** 12

Second: Perform all additions and subtractions last. The results of the two multiplications are subtracted from each other. Since both of the multiplications are good to the second decimal place, the final result is rounded to the second decimal place.

5.3371 – **4.465** 12 = **0.871** 92

Therefore the answer is rounded to **0.87.**

EXERCISE:

59. In the following mixed calculations perform multiplications and divisions before doing the additions and subtractions. Keep track of the number of significant figures at each stage of a calculation.
    - (a) 25.00 x 0.1000 – 15.87 x 0.1036
    - (b) 35.0 x 1.525 + 50.0 x 0.975
    - (c) (0.865 – 0.800) x (1.593 + 9.04)
    - (d) $\dfrac{(0.3812 + 0.4176)}{(0.0159 - 0.0146)}$
    - (e) $\dfrac{3.65}{0.3354} - \dfrac{6.14}{0.1766}$
    - (f) $\dfrac{5.3 \times 0.1056}{0.1036 - 0.0978}$
    - (g) (0.341 x 18.64 – 6.00) x 3.176
    - (h) 9.34 x 0.071 46 – 6.88 x 0.081 15

UNIT III : THE PHYSICAL PROPERTIES AND PHYSICAL CHANGES OF SUBSTANCES

III.1. SOME BASIC DEFINITIONS IN SCIENCE

In this course you will be asked to describe substances in many different ways. To make sure that we are using a common vocabulary, some important terms first must be defined and agreed upon.

Definitions: **QUALITATIVE** information is ***NON–NUMERICAL*** information.

QUANTITATIVE information is ***NUMERICAL*** information.

Example	Qualitative Description	Quantitative Description
Your height	tall, short	5' 10" , 180 cm
Your weight	normal, heavy	110 lb, 123 kg
Chemistry 11 mark	awesome, fail	100%, 55/200

(Qualitative and quantitative information serve different purposes. "The boy is five feet tall" is a quantitative statement which makes no judgement about the boy's height. If the boy was only six years old, the qualitative statement "the boy was extraordinarily tall" would let us know that an unusual situation existed.)

An **OBSERVATION** is ***qualitative*** information collected through the direct use of our senses.

An **INTERPRETATION** (or "inference") is an attempt to put meaning into an observation.

A **DESCRIPTION** is a ***list*** of the properties of something.

DATA is ***quantitative*** information which is experimentally–determined or obtained from references.

An **EXPERIMENT** is a test or a procedure that is carried out in order to discover a result.

A **HYPOTHESIS** is a SINGLE, UNPROVEN assumption or idea which attempts to explain why nature behaves in a specific manner. When initially put forward, hypotheses are tentative but, if they survive testing, eventually gain general acceptance.

A **THEORY** is a set of hypotheses that ties together a large number of observations of the real world into a logically consistent and understandable pattern. In other words, a theory is a TESTED, REFINED and EXPANDED explanation of why nature behaves in a given way.

A **LAW** is a broad generalization or summary statement which describes a large amount of experimental evidence stating how nature behaves when a particular situation occurs.

Some Additional Comments about Hypotheses, Theories and Laws

The following are general characteristics of HYPOTHESES.

- Hypotheses are normally single assumptions.
- Hypotheses are narrow in their scope of explanation.
- When originally proposed, hypotheses are tentative (being based on suggestive but VERY incomplete evidence) but may become generally accepted after more complete testing.

The following are general characteristics of THEORIES.

- Theories are composed of one or more underlying hypotheses.
- Theories are broad in scope and may have subtle implications which are not foreseen when they are proposed because they provide explanations for entire "fields" of related behaviour.